### Wisconsin Content Standards - Math 167

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

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<tr>
<th>In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.</th>
<th>In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.</th>
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<td>The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.</td>
<td>The ability of students to use graphing calculators to investigate functions and series is assessed by homework.</td>
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<td>Facilitating the building of student conceptual and procedural understanding.</td>
<td>Students’ understanding of calculus concepts from algebraic, numerical, and graphical perspectives is assessed on tests. Students’ ability to follow a strategy for evaluating integrals and a strategy for determining convergence of series is also measured on exams.</td>
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| Helping all students build understanding of the discipline including:  
  • Confidence in their abilities to utilize mathematical knowledge.  
  • Awareness of the usefulness of mathematics.  
  • The economic implications of fine mathematical preparation. | Students’ confidence in integrating their knowledge from previous courses in Algebra, Trigonometry and Calculus with the new calculus they are learning is assessed by assignments. Test questions measure students awareness of applications of exponential functions and separable differential equations to the sciences. |
<p>| Exploring, conjecturing, examining and testing all aspects of problem solving. | Students’ skill at exploring and make conjectures about the convergence or divergence of series is measured through assigned problems. A student's ability to explore different integration techniques to determine which technique will work for a particular integral is assessed by a number of homework problems. |
| Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations. | Students’ talent at evaluating a single integral using different integration techniques and determining the convergence of a single series using different tests for convergence is measured through homework problems. Assessment of students’ ability to use problem solving approaches effectively and to model real-world situations is done by assigned problems. |
| Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof. | Students’ skill at making convincing mathematical arguments about the convergence or divergence of series is measured by assigned homework. |
| Expressing ideas orally, in writing, and visually- using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts. | Students’ ability in expressing their understanding of calculus concepts algebraically, numerically and graphically is appraised on written tests and assignments. |
| Connecting the concepts and procedures of mathematics, drawing connections between | Students awareness of connections between the general idea of an inverse function in a calculus |</p>
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<th>Mathematical strands, between mathematics and other disciplines, and with daily life.</th>
<th>context and the specific examples of these functions, logarithms, arcsine, and arctangent, is measured in assignments. How well a student makes connections between the procedure of testing series and the theories behind the tests for convergence is assessed in homework.</th>
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<td>Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.</td>
<td>Students' ability to solve problems using algebraic, graphical and numeric approaches is appraised using assigned problems. It is often necessary to choosing one approach to understand the question, another to solve the problem and perhaps, a third to demonstrate the result. The ability to do this is measured by assigning problems that require evaluating limits and definite integrals. A fourth way to represent functions is with Taylor series. Students' talent at using Taylor series representations to solve integration and limit problems is assessed by additional homework.</td>
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| Mathematical processes including:  
• Problem solving.  
• Communication.  
• Reasoning and formal and informal argument.  
• Mathematical connections.  
• Representations.  
• Technology. | Tests and assignments assess the ability of a student to solve problems; communicate their results mathematically; give formal justifications of their reasoning; make connections between theory and example; make connections between algebraic, graphical and numerical viewpoints of a question; represent functions algebraically, numerically, graphically and with series; and use graphing calculators to study functions and series. |
| Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including:  
• Number sense.  
• Set theory.  
• Number and operation.  
• Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics.  
• Number systems through the real numbers, their properties and relations.  
• Computational procedures.  
• Proportional reasoning.  
• Number theory. | Students' skill at approximating definite integrals and limits numerically and their understanding of the real numbers through infinite series and series representations of decimals is measured by both exam questions and homework problems. |
Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including:

- Advanced counting procedures, including union and intersection of sets, and parenthetical operations.
- Algebraic and transcendental numbers.
- The complex number system, including polar coordinates.
- Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs.
- Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated.
- Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).

Assessment of students’ ability to work in polar coordinates in solving calculus problems, to graph functions, to calculate areas, to integrate numerically, to find tangent lines and to calculate arc lengths in polar coordinates is done through a selection of homework problems.
| Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including:  
• Formal and informal argument.  
• Names, properties, and relationships of two- and three-dimensional shapes.  
• Spatial sense.  
• Spatial reasoning and the use of geometric models to represent, visualize, and solve problems.  
• Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships.  
• Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system.  
• Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools.  
• The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money.  
• Measuring, estimating, and using measurement to describe and compare geometric phenomena.  
• Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems. |
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<td>A combination of exam questions and homework problems are used to assess each student's ability to compare rectangular and polar coordinates and to use calculus to measure areas and arc lengths.</td>
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| Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including:  
• Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry.  
• Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations.  
• Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations. |
| Not assessed in this course. |
| Statistics and probability from both abstract and Some use of data is incorporated when students |
concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including:
• Use of data to explore real-world issues.
• The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data.
• Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance.
• Analysis and interpretation of data.
• Randomness, sampling, and inference.
• Probability as a way to describe chances or risk in simple and compound events.
• Outcome prediction based on experimentation or theoretical probabilities.

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<th>Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including:</th>
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<td>• Use of the random variable in the generation and interpretation of probability distributions.</td>
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<td>• Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation.</td>
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<td>• Probability theory and its link to inferential statistics.</td>
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<td>• Discrete and continuous probability distributions as bases for inference.</td>
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<td>• Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.</td>
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<th>Not assessed in this course.</th>
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Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including:

- Patterns.
- Functions as used to describe relations and to model real world situations.
- Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships.
- Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another.
- Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions.
- Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods.
- Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.

Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including:

- Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series.
- Modeling to solve problems.
- Calculus techniques including finding limits, derivatives, integrals, and using special rules.
- Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass.
- Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization.
- Multivariate calculus.
- Differential equations.

Assigned homework and exam problems are used to assess how well each student can handle the following.

1. Identifying patterns.
2. Using functions to describe real world exponential growth and decay problems.
3. Representing functions in four ways: algebraically, graphically, numerically and as Taylor series; and convert from one representation to another.
4. Understanding polynomials through Taylor series.
5. Limits, derivatives and integrals of polynomial, rational, trigonometric, algebraic and exponential functions.
6. Working with the inverses of the functions listed in 5 from an algebraic, graphical and numerical perspective.
7. The underlying concepts of calculus, including, rate of change, limits, and approximations for irregular areas.

Assigned homework and exam problems are used to ascertain how well each student understands each of the following.

1. The concepts of calculus, including limits (from an intuitive viewpoint), tangents, derivatives, sequences and series.
2. Using modeling to solve exponential growth and decay problems.
3. Using calculus techniques to find limits, derivatives, and integrals.
4. Applying calculus to the problems of area, exponential growth and decay, and arc length.
| Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including:  
  • Counting techniques.  
  • Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks.  
  • Iteration and recursion. | Not assessed in this course. |
|---|---|
| Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including:  
  • Topics, including symbolic logic, induction, linear programming, and finite graphs.  
  • Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems.  
  • Developing and analyzing algorithms. | Not assessed in this course. |