Wisconsin Content Standards - Math 456/656 **APPENDIX C**

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.	**In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.**
The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.	On tests and in assignments students show that they are able to the use technology to simulate Markov Chains and other random processes. The ability of students to use the models studied in this course, which are classical and frame the basic questions that are at the heart of the development of modern probability theory, is measured using a combination of exam questions and homework problems. Included in this assessment is the use of models that arise frequently in the mathematical formulation of problems encountered in real-world settings.
Facilitating the building of student conceptual and procedural understanding.	Markov chains and other basic random processes that are studied in this course have both well-defined procedural solutions and an underlying mathematical theory that lends itself to a conceptual understanding of these solutions. Conceptual understanding is assessed by assignments and both homework and exams assess procedural understanding.
Helping all students build understanding of the discipline including: • Confidence in their abilities to utilize mathematical knowledge. • Awareness of the usefulness of mathematics. • The economic implications of fine mathematical preparation.	The problems on homework assignments and exams assess how well students are able to integrate a rich background of material from Calculus and Discrete Mathematics. Students are asked to solve a number of “real-world” problems that can be formulated in terms of Markov Chains and other basic random processes that are studied in this course. The problems that they work on come from queuing theory and other topics in this course that arise in the form of optimization problems which on a larger scale, such as is encountered in industry, are of considerable economic importance.
Exploring, conjecturing, examining and testing all aspects of problem solving.	A strong emphasis is placed on measuring problem-solving skills in the homework assigned for this course. The use of computers to simulate random processes yields important opportunities to assess students' ability to carry out the multiple stages of active problem solving.

Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations.	Assigned problems are used to check how well students are at determining steady-state solutions for Markov Chains using both recurrence relations and matrix operations. Homework is also used to see if students are able to determine recurrence times and the times needed to reach certain ergodic states using generalizations of geometric random variables and using simulation by a computer of the accompanying random processes.
Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof.	Students are required to reproduce on exams the proof of certain classical theorems, i.e., those that characterize certain process as Poisson processes based on their being time invariant, etc.
Expressing ideas orally, in writing, and visually-, using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts.	Written mathematical skills are measured by having students regularly submit homework assignments and take exams. In-class participation is used to measure oral mathematical skills.
Connecting the concepts and procedures of mathematics, drawing connections between mathematical strands, between mathematics and other disciplines, and with daily life.	Students' grasp of how the mathematical models encountered in this course are used in the study of gambling and the growth of populations is assessed by exams and homework.
Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.	How skilled students are at taking the models encountered in Advanced Probability and translating them into terms that allow solutions by using calculus or discrete mathematics is assessed by homework problems and exam questions.
Mathematical processes including: • Problem solving. • Communication. • Reasoning and formal and informal argument. • Mathematical connections. • Representations. • Technology.	In class work and out-of-class assignments measure the development of problem solving and communication skills on the part of the student. This assessment includes problems whose solutions require the mathematical concepts from any one of a number of other mathematics courses in the curriculum: discrete mathematics, calculus, linear algebra, mathematical programming, differential equations, and abstract algebra.

Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including: • Number sense. • Set theory. • Number and operation. • Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics. • Number systems through the real numbers, their properties and relations. • Computational procedures. • Proportional reasoning. • Number theory.	None.
Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including: • Advanced counting procedures, including union and intersection of sets, and parenthetical operations. • Algebraic and transcendental numbers. • The complex number system, including polar coordinates. • Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs. • Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated. • Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).	Primarily through assigned problems and to a lesser degree exam questions, students are assessed on their ability to do the following. 1. Using advanced counting techniques in enumerating the number of ways in which a given event can occur. 2. Reformulating methods of enumeration in terms of linear algebra and matrix operations. 3. Intuiting values, from a basic consideration of the qualitative properties of the process, that answer queries about the random processes that we study. 4. Deciding on the reasonableness of results. 5. Using technology with linear algebra, matrix methods, and differential equations to carry out computations.

Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including: • Formal and informal argument. • Names, properties, and relationships of two- and three-dimensional shapes. • Spatial sense. • Spatial reasoning and the use of geometric models to represent, visualize, and solve problems. • Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships. • Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system. • Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools. • The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money. • Measuring, estimating, and using measurement to describe and compare geometric phenomena. • Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems.	Assigned problems are used to assess how well students can do the following in the study of two-dimensional random walks. 1. Identifying the arrival times for certain symmetrically placed sites as being equal. 2. The use of symmetry to reduce the size of the state space that must be employed in the construction of equations for the arrival times of the random walk on the sites of a lattice.

Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including: • Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry. • Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations. • Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations.	None.
Statistics and probability from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including: • Use of data to explore real-world issues. • The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data. • Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance. • Analysis and interpretation of data. • Randomness, sampling, and inference. • Probability as a way to describe chances or risk in simple and compound events. • Outcome prediction based on experimentation or theoretical probabilities.	None.

Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including: • Use of the random variable in the generation and interpretation of probability distributions. • Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation. • Probability theory and its link to inferential statistics. • Discrete and continuous probability distributions as bases for inference. • Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.	The ability to use random variables, such as the state of a Markov chain, as well as the arrival and recurrence times for the same, is measured by homework and exams.
Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Patterns. • Functions as used to describe relations and to model real world situations. • Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships. • Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another. • Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions. • Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods. • Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.	Both homework and exams assess the ability of students to calculate recurrence times, first arrival times, and occupation probabilities by solving systems of equations.

Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including: • Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series. • Modeling to solve problems. • Calculus techniques including finding limits, derivatives, integrals, and using special rules. • Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass. • Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization. • Multivariate calculus. • Differential equations.	Selected homework problems gauge how well students are able to do the following. 1. Find, by ergodic theory, the equilibrium state of a Markov Chain as the limit of the chain's behavior as the time parameter tends to infinity. 2. Formulate basic problems in gambling theory and population dynamics in terms of Markov chains and other random processes. 3. Using communicating classes of a Markov Chain to form an equivalence class on the state space of the chain.
Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Counting techniques. • Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks. • Iteration and recursion.	The following are assessed using both homework and exams. 1. Using counting techniques to identify the number of ways in which an event can occur and using these enumerations to calculate certain probabilities that arise in a natural way in the study of Markov Chains and other models. 2. Representing Markov chains in terms of weighted, directed graphs by drawing edges between communicating states and employing the corresponding transition probability between these states as the weight. 3. Determining recurrence and arrival times by formulating and solving certain recurrence relations.
Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including: • Topics, including symbolic logic, induction, linear programming, and finite graphs. • Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems. • Developing and analyzing algorithms.	None.