

Wisconsin Content Standards – Math 436

APPENDIX C

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

<p>In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.</p>	<p>In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.</p>
<p>The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.</p>	<p>This course is all about the historical roots of what students will be teaching to their secondary and middle school pupils. Two major tests are used to assess the students overall knowledge in the area of math history.</p>
<p>Facilitating the building of student conceptual and procedural understanding.</p>	<p>The journal kept by students for the course is used to assess students' understanding of the struggle made by preeminent minds in the advancement of mathematics and their appreciation of the true nature of the subject and its human roots.</p>
<p>Helping all students build understanding of the discipline including:</p> <ul style="list-style-type: none"> • Confidence in their abilities to utilize mathematical knowledge. • Awareness of the usefulness of mathematics. • The economic implications of fine mathematical preparation. 	<p>Students ability (or perceived deficiency) to utilize mathematical knowledge and their realization that the creators (discoverers) of mathematics sometimes lacked confidence in their own work is assessed through their journal entries.</p>
<p>Exploring, conjecturing, examining and testing all aspects of problem solving.</p>	<p>Students' understanding of how exploring, conjecturing and analyzing were used for the three major unsolvable construction problems of Greek mathematics (and how results obtained are not necessarily ones that were sought) is appraised through exam questions.</p>
<p>Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations.</p>	<p>None.</p>
<p>Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof.</p>	<p>To assess this, students are asked to keep a journal of mathematical readings (150 pages). Half of each journal entry is a reflection on the reading.</p>
<p>Expressing ideas orally, in writing, and visually-, using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts.</p>	<p>Students are assessed on their skill in writing mathematics through two 4 – 6 page papers on some aspect of mathematical history. Students' oral skills in mathematics are measured using two brief, required, 8 to 10 minute presentations on a mathematical topic.</p>

<p>Connecting the concepts and procedures of mathematics, drawing connections between mathematical strands, between mathematics and other disciplines, and with daily life.</p>	<p>None.</p>
<p>Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.</p>	<p>None.</p>
<p>Mathematical processes including:</p> <ul style="list-style-type: none"> • Problem solving. • Communication. • Reasoning and formal and informal argument. • Mathematical connections. • Representations. • Technology. 	<p>Students' ability to use a variety of formal and informal logical arguments is appraised by exam questions.</p>
<p>Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including:</p> <ul style="list-style-type: none"> • Number sense. • Set theory. • Number and operation. • Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics. • Number systems through the real numbers, their properties and relations. • Computational procedures. • Proportional reasoning. • Number theory. 	<p>Exam questions and in-class discussions are used to do assessment of students' understanding of the development of viable number systems including irrational numbers and how to prove that numbers are irrational.</p>

Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including:

- Advanced counting procedures, including union and intersection of sets, and parenthetical operations.
- Algebraic and transcendental numbers.
- The complex number system, including polar coordinates.
- Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs.
- Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated.
- Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).

Exam questions are used to ascertain students' grasp of the complex number system in conjunction with Hamilton's quaternions and the difference between algebraic and transcendental numbers.

Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including:

- Formal and informal argument.
- Names, properties, and relationships of two- and three-dimensional shapes.
- Spatial sense.
- Spatial reasoning and the use of geometric models to represent, visualize, and solve problems.
- Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships.
- Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system.
- Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools.
- The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money.
- Measuring, estimating, and using measurement to describe and compare geometric phenomena.
- Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems.

How well students comprehend early attempts at measurement of area and volume and some results influenced by the introduction of coordinate geometry is measured by exam questions.

Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including:

- Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry.
- Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations.
- Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations.

The development of Euclidean geometry as an axiomatic mathematical system is a major component of the first half of the course. The rise of non-Euclidean geometries is discussed in the second half of the course. Student assessment is accomplished by test questions on the mid-term and final.

Statistics and probability from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including:

- Use of data to explore real-world issues.
- The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data.
- Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance.
- Analysis and interpretation of data.
- Randomness, sampling, and inference.
- Probability as a way to describe chances or risk in simple and compound events.
- Outcome prediction based on experimentation or theoretical probabilities.

The ability of students to work with some elementary probability problems which lie at the foundation of probability (for example, problem of points) is appraised via exam problems.

Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including:

- Use of the random variable in the generation and interpretation of probability distributions.
- Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation.
- Probability theory and its link to inferential statistics.
- Discrete and continuous probability distributions as bases for inference.
- Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.

None.

Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including:

- Patterns.
- Functions as used to describe relations and to model real world situations.
- Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships.
- Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another.
- Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions.
- Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods.
- Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.

Journal entries and exam questions are used to measure students' understanding of the explicit solution to cubic equations, the emergence of imaginary numbers from the study of cubic equations, and how mathematicians advance the body of knowledge sometimes without a complete understanding of the nature of the problems they are attempting to solve.

<p>Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including:</p> <ul style="list-style-type: none"> • Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series. • Modeling to solve problems. • Calculus techniques including finding limits, derivatives, integrals, and using special rules. • Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass. • Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization. • Multivariate calculus. • Differential equations. 	<p>Students are assessed by test questions and journal entries on their grasp of the historical development of calculus including early attempts at defining the derivative, finding slopes of tangent lines, and early methods of integration.</p>
<p>Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including:</p> <ul style="list-style-type: none"> • Counting techniques. • Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks. • Iteration and recursion. 	<p>None.</p>
<p>Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including:</p> <ul style="list-style-type: none"> • Topics, including symbolic logic, induction, linear programming, and finite graphs. • Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems. • Developing and analyzing algorithms. 	<p>Students' ability to use some elementary algorithms (for example, Heron's way of computing square roots, ways of multiplying numbers using log or trig tables) is gauged through questions on exams.</p>