Wisconsin Content Standards – Math 431 **APPENDIX C**

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.	**In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.**
The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.	Students are introduced to the idea of a generalized function (or distribution). Students use graphing calculators and/or computer algebra systems to help them solve systems of differential equations.
Facilitating the building of student conceptual and procedural understanding.	Students are tested on their understanding of the theories of ordinary differential equations. Students are tested on their ability to follow procedures to solve and analyze differential equations.
Helping all students build understanding of the discipline including: • Confidence in their abilities to utilize mathematical knowledge. • Awareness of the usefulness of mathematics. • The economic implications of fine mathematical preparation.	Students integrate their knowledge from previous courses in calculus, differential equations and linear algebra with the differential equations topics they are learning, gaining confidence in their ability to utilize mathematical knowledge. Students investigate applications of differential equations to the sciences.
Exploring, conjecturing, examining and testing all aspects of problem solving.	Students explore and make conjectures when determining which technique might work to solve a differential equation. Students explore and make conjectures about convolutions of functions and Fourier series. Students use phase plane analysis to explore the behavior of solutions to differential equations.
Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations.	Students compare solutions to differential equations found using different techniques. For example, students solve linear systems of differential equations using the method of elimination and using matrix theory. Students are tested on their ability to use problem solving approaches effectively. Some real-world applications included are heat flow, vibrating strings, and steady state phenomena.
Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof.	Students make convincing mathematical arguments about the behavior of solutions of differential equations. Students create proofs about the properties of Fourier series and convolutions of functions.

Expressing ideas orally, in writing, and visually-, using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts.	Students express their understanding of the theory and solutions of ordinary and partial differential equations using mathematical language, notation and symbolism on written tests and assignments. Students give an oral report on a special topic in differential equations, which they investigate independently with a classmate.
Connecting the concepts and procedures of mathematics, drawing connections between mathematical strands, between mathematics and other disciplines, and with daily life.	Students make connections between the theory of ordinary and partial differential equations and the solutions to ordinary and partial differential equations. Students make connections between Fourier series and Taylor series. Students make connections between differential equations and the sciences.
Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.	Students translate between systems of linear differential equations and systems of linear algebraic equations, exploiting their knowledge of algebra to solve differential equations. Students represent solutions algebraically, graphically, and with series. Students are introduced to representing qualitative solutions to differential equations via phase plane analysis.
Mathematical processes including: • Problem solving. • Communication. • Reasoning and formal and informal argument. • Mathematical connections. • Representations. • Technology.	On tests and assignments, students solve problems; communicate their results mathematically; give formal justifications of their reasoning; make connections between theory and example; make connections between solutions found using multiple approaches; represent solutions to differential equations algebraically, graphically, with Taylor's series and with Fourier series; and use graphing calculators or computer algebra systems to study the behavior of solutions.
Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including: • Number sense. • Set theory. • Number and operation. • Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics. • Number systems through the real numbers, their properties and relations. • Computational procedures. • Proportional reasoning. • Number theory.	Students use their number sense, and understanding of the real number system on their homework assignments and tests.

Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including: • Advanced counting procedures, including union and intersection of sets, and parenthetical operations. • Algebraic and transcendental numbers. • The complex number system, including polar coordinates. • Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs. • Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated. • Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).	Students use complex numbers in their solutions of systems of differential equations, with Euler's formula to translate complex solutions into real solutions. Students use complex numbers to understand Fourier series.

Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including: • Formal and informal argument. • Names, properties, and relationships of two- and three-dimensional shapes. • Spatial sense. • Spatial reasoning and the use of geometric models to represent, visualize, and solve problems. • Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships. • Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system. • Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools. • The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money. • Measuring, estimating, and using measurement to describe and compare geometric phenomena. • Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems.	Students use spatial sense to interpret the behavior of solutions to differential equations. Students use spatial sense to understand phase plane diagrams and stability issues.

Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including: • Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry. • Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations. • Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations.	Students use vectors to solve linear systems of equations.
Statistics and probability from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including: • Use of data to explore real-world issues. • The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data. • Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance. • Analysis and interpretation of data. • Randomness, sampling, and inference. • Probability as a way to describe chances or risk in simple and compound events. • Outcome prediction based on experimentation or theoretical probabilities.	Not assessed in this course.

Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including: • Use of the random variable in the generation and interpretation of probability distributions. • Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation. • Probability theory and its link to inferential statistics. • Discrete and continuous probability distributions as bases for inference. • Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.	Not assessed in this course.
Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Patterns. • Functions as used to describe relations and to model real world situations. • Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships. • Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another. • Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions. • Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods. • Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.	Students review concepts of calculus as necessary. Students find patterns in families of ordinary and partial differential equations and their solutions. Students model real world situations with differential equations and describe their solutions with functions. Students represent solutions to differential equations algebraically, graphically, as Taylor's series and as Fourier series; they compare and contrast these representations. Students use the properties of polynomial, rational, trigonometric, algebraic and exponential functions in their solutions to differential equations.

Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including: • Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series. • Modeling to solve problems. • Calculus techniques including finding limits, derivatives, integrals, and using special rules. • Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass. • Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization. • Multivariate calculus. • Differential equations.	Students demonstrate knowledge of the concepts of calculus, including limits (from an intuitive viewpoint), tangents, derivatives and series. Students use modeling to solve real-world problems such as heat flow, vibrating strings, and steady state phenomena. Students use calculus techniques to find derivatives and integrals. Students solve linear systems of differential equations using the method of elimination, matrix theory, and Laplace transforms; they compare and contrast these approaches. Students use separation of variables and Fourier series to find solutions to partial differential equations.
Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Counting techniques. • Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks. • Iteration and recursion.	Not assessed in this course.
Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including: • Topics, including symbolic logic, induction, linear programming, and finite graphs. • Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems. • Developing and analyzing algorithms.	Students use matrices to represent systems of differential equations. Students use eigenvalue and eigenvector theory to solve systems of differential equations.