

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

| <p><b>In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.</b></p>   | <p><b>In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.</b></p>  |
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| <p>The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.</p>  | <p>As a research course students will produce a paper on their research results. Depending on the research topic selected, the paper may include structures within mathematics, the history of an area of mathematics, the evolution of mathematics or how mathematics interacts with a particular technology.</p>  |
| <p>Facilitating the building of student conceptual and procedural understanding.</p>  | <p>The paper written by the student will allow assessment of the student's understanding of the concepts and procedures related to the research topic.</p>  |
| <p>Helping all students build understanding of the discipline including:</p> <ul style="list-style-type: none"> <li>• Confidence in their abilities to utilize mathematical knowledge.</li> <li>• Awareness of the usefulness of mathematics.</li> <li>• The economic implications of fine mathematical preparation.</li> </ul> | <p>The paper written on the research results will showcase the student's skill in using the mathematics they have learned. Since the research usually includes working with a company on an applied mathematics topic, the paper normally includes a discussion of how mathematics is used in the solution of a real-world problem and the economic impact this has on the company.</p> |
| <p>Exploring, conjecturing, examining and testing all aspects of problem solving.</p>   | <p>The research project paper will have the student describe alternatives explored, conjectures made and tested, and what conclusions they drew.</p>  |
| <p>Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations.</p>  | <p>Depending on the research done, students may need to consider alternative strategies, may need to develop mathematical models and may need to go from specific to general solutions. The paper would explore what was considered here how the alternatives were evaluated.</p>   |
| <p>Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof.</p>   | <p>The student will need to formulate appropriate mathematical questions as part of their research. They will need to make conjectures and prove or disprove them. The paper should list the conjectures made, the counter-examples constructed and the proofs constructed as part of this research.</p>  |
| <p>Expressing ideas orally, in writing, and visually-, using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts.</p>   | <p>Students may give an oral report on their research in addition to the written report.</p>  |

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| <p>Connecting the concepts and procedures of mathematics, drawing connections between mathematical strands, between mathematics and other disciplines, and with daily life.</p>  | <p>Depending on the topic studied, the research may connect mathematical concepts with their use in other disciplines. The paper would include these connections.</p>  |
| <p>Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.</p>   | <p>As a research project, the solution of the problem may require selecting the appropriate representation or may require converting among representations to complete the research. The paper will discuss these representations and conversions.</p> |
| <p>Mathematical processes including:</p> <ul style="list-style-type: none"> <li>• Problem solving.</li> <li>• Communication.</li> <li>• Reasoning and formal and informal argument.</li> <li>• Mathematical connections.</li> <li>• Representations.</li> <li>• Technology.</li> </ul>   | <p>The research project would normally include problem solving techniques and communication as well as perhaps the other mathematical processes. The paper would discuss the processes used.</p>   |
| <p>Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including:</p> <ul style="list-style-type: none"> <li>• Number sense.</li> <li>• Set theory.</li> <li>• Number and operation.</li> <li>• Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics.</li> <li>• Number systems through the real numbers, their properties and relations.</li> <li>• Computational procedures.</li> <li>• Proportional reasoning.</li> <li>• Number theory.</li> </ul> | <p>Depending on the research subject, the use of number operations and relationships would be included in the paper.</p>   |

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| <p>Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including:</p> <ul style="list-style-type: none"><li>• Advanced counting procedures, including union and intersection of sets, and parenthetical operations.</li><li>• Algebraic and transcendental numbers.</li><li>• The complex number system, including polar coordinates.</li><li>• Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs.</li><li>• Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated.</li><li>• Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).</li></ul> | <p>If a teaching number operations and relationships is selected for the research topic, the paper would cover one or more of these areas.</p> |
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| <p>Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including:</p> <ul style="list-style-type: none"> <li>• Formal and informal argument.</li> <li>• Names, properties, and relationships of two- and three-dimensional shapes.</li> <li>• Spatial sense.</li> <li>• Spatial reasoning and the use of geometric models to represent, visualize, and solve problems.</li> <li>• Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships.</li> <li>• Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system.</li> <li>• Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools.</li> <li>• The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money.</li> <li>• Measuring, estimating, and using measurement to describe and compare geometric phenomena.</li> <li>• Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems.</li> </ul> | <p>If a geometry topic is selected for the research topic, the paper would cover one or more of these areas.</p> |
| <p>Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including:</p> <ul style="list-style-type: none"> <li>• Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry.</li> <li>• Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations.</li> <li>• Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations.</li> </ul>  | <p>If a teaching geometry is selected for research the paper would cover one or more of these areas.</p>         |

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| <p>Statistics and probability from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including:</p> <ul style="list-style-type: none"> <li>• Use of data to explore real-world issues.</li> <li>• The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data.</li> <li>• Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance.</li> <li>• Analysis and interpretation of data.</li> <li>• Randomness, sampling, and inference.</li> <li>• Probability as a way to describe chances or risk in simple and compound events.</li> <li>• Outcome prediction based on experimentation or theoretical probabilities.</li> </ul> | <p>The paper would include a discussion of one or more of these areas if a topic that includes statistics or probability is selected for research.</p> |
| <p>Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including:</p> <ul style="list-style-type: none"> <li>• Use of the random variable in the generation and interpretation of probability distributions.</li> <li>• Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation.</li> <li>• Probability theory and its link to inferential statistics.</li> <li>• Discrete and continuous probability distributions as bases for inference.</li> <li>• Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.</li> </ul>  | <p>The paper would include one or more of these areas if teaching statistics or probability is the research topic.</p>                                 |

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| <p>Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including:</p> <ul style="list-style-type: none"> <li>• Patterns.</li> <li>• Functions as used to describe relations and to model real world situations.</li> <li>• Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships.</li> <li>• Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another.</li> <li>• Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions.</li> <li>• Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods.</li> <li>• Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.</li> </ul> | <p>Depending on the research selected, the paper may include these topics.</p>                         |
| <p>Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including:</p> <ul style="list-style-type: none"> <li>• Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series.</li> <li>• Modeling to solve problems.</li> <li>• Calculus techniques including finding limits, derivatives, integrals, and using special rules.</li> <li>• Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass.</li> <li>• Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization.</li> <li>• Multivariate calculus.</li> <li>• Differential equations.</li> </ul>  | <p>If the research involved teaching these subjects, this material would be included in the paper.</p> |

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| <p>Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including:</p> <ul style="list-style-type: none"> <li>• Counting techniques.</li> <li>• Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks.</li> <li>• Iteration and recursion.</li> </ul>                           | <p>If the research involves the use of discrete mathematics, the paper would include a discussion of how discrete processes were used.</p> |
| <p>Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including:</p> <ul style="list-style-type: none"> <li>• Topics, including symbolic logic, induction, linear programming, and finite graphs.</li> <li>• Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems.</li> <li>• Developing and analyzing algorithms.</li> </ul> | <p>If the research involves how discrete mathematics is taught, the paper would discuss how this was done.</p>                             |