Wisconsin Content Standards – Math 347 **APPENDIX C**

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.	**In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.**
The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.	Since the theme of the course is the use of mathematics to analyze algorithms which are in general implemented on computers, the interaction between technology and math is heavily emphasized.
Facilitating the building of student conceptual and procedural understanding.	Students are tested over both how and why certain algorithms solve or don't solve given problems.
Helping all students build understanding of the discipline including: • Confidence in their abilities to utilize mathematical knowledge. • Awareness of the usefulness of mathematics. • The economic implications of fine mathematical preparation.	A large part of the course involves the application of concepts learned in calculus, discrete math, and linear algebra to the analysis of algorithms thereby building confidence in this previously encountered knowledge.
Exploring, conjecturing, examining and testing all aspects of problem solving.	Some exploring is done as different inputs are provided to programs in homework assignments. Hypotheses to various theorems are examined by testing alternatives.
Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations.	Some problems (solving differential equations) are approached through a variety of methods. Some real-world applications are found in problem sets.
Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof.	Principal questions such as: why does an algorithm work, how does it work, how efficient is it, how accurate is it, are posed and given informal and formal proofs. Constructing limiting examples and counter-examples are assessed through homework.
Expressing ideas orally, in writing, and visually-, using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts.	Homework and tests provide assessment of ability to use mathematical language.
Connecting the concepts and procedures of mathematics, drawing connections between mathematical strands, between mathematics and other disciplines, and with daily life.	Some algorithms are directly related to other sciences (ordinary and partial differential equations).

Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.	The course constantly explores the interplay between geometric and algebraic descriptions of an algorithm. Algorithms that solve boundary-value problems are good examples of this interaction.
Mathematical processes including: • Problem solving. • Communication. • Reasoning and formal and informal argument. • Mathematical connections. • Representations. • Technology.	Homework problems are of two types: exercise and problem-solving. Students are required to justify answers through informal arguments. Programs implemented in the form of Mathematica notebooks and computers must be utilized to solve many problems.
Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including: • Number sense. • Set theory. • Number and operation. • Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics. • Number systems through the real numbers, their properties and relations. • Computational procedures. • Proportional reasoning. • Number theory.	None.
Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including: • Advanced counting procedures, including union and intersection of sets, and parenthetical operations. • Algebraic and transcendental numbers. • The complex number system, including polar coordinates. • Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs. • Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated. • Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).	Complex numbers and basic complex arithmetic are reviewed in discussing Fast Fourier Transforms. The numerical solution to integral equations requires approximation techniques. Numeric algorithms are coded in Mathematica. Students are required to utilize existing programs or write their own. Reasonableness of computed answers and error bounds are discussed extensively.

Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including: • Formal and informal argument. • Names, properties, and relationships of two- and three-dimensional shapes. • Spatial sense. • Spatial reasoning and the use of geometric models to represent, visualize, and solve problems. • Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships. • Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system. • Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools. • The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money. • Measuring, estimating, and using measurement to describe and compare geometric phenomena. • Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems.	Some aspects of analytic geometry are assessed in homework and test. Students are introduced to classic partial differential equation problems through their physical models. This requires a good understanding of coordinate geometry systems and spatial reasoning.

Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including: • Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry. • Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations. • Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations.	Informal proofs, arguments, and counter-examples are assessed through homework, classwork, and tests.
Statistics and probability from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including: • Use of data to explore real-world issues. • The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data. • Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance. • Analysis and interpretation of data. • Randomness, sampling, and inference. • Probability as a way to describe chances or risk in simple and compound events. • Outcome prediction based on experimentation or theoretical probabilities.	Some “real-world” data used in homework problems, ordinary and partial differential equations, signal processing.

Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including: • Use of the random variable in the generation and interpretation of probability distributions. • Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation. • Probability theory and its link to inferential statistics. • Discrete and continuous probability distributions as bases for inference. • Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.	None.
Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Patterns. • Functions as used to describe relations and to model real world situations. • Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships. • Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another. • Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions. • Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods. • Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.	Solution of equations, system of equations, linear and non-linear, are major components of the course. Limits and approximation techniques are used throughout the course. Polynomials and trigonometric functions are used as approximations to functions.

Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including: • Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series. • Modeling to solve problems. • Calculus techniques including finding limits, derivatives, integrals, and using special rules. • Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass. • Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization. • Multivariate calculus. • Differential equations.	Concepts of calculus reviewed and applied. Intermediate value theorem. Mean-value Theorem Derivatives Definite integrals Convergence of sequences. Taylor series. Numerical approximation Some multivariable calculus is systems of non-linear equations and least-square fits.
Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Counting techniques. • Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks. • Iteration and recursion.	Iteration and recursion are major components in solving equations, systems of linear and nonlinear equations. Some counting techniques are used in computing number of operations to implement an algorithm.
Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including: • Topics, including symbolic logic, induction, linear programming, and finite graphs. • Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems. • Developing and analyzing algorithms.	Matrices are used extensively in solving systems of equations and signal processing. Developing and analyzing algorithms are major components of the course.