Wisconsin Content Standards - Math 331 **APPENDIX C**

All professional education content courses leading to certification shall include teaching and assessment of the Wisconsin Content Standards in the content area.

In this column, list the Wisconsin Content Standards that are included in this course. The Standards for each content area are found in the Wisconsin Content Standards document.	**In this column, indicate the nature of the performance assessments used in this course to evaluate student proficiency in each standard.**
The structures within the discipline, the historical roots and evolving nature of mathematics, and the interaction between technology and the discipline.	A student's skill in using graphing calculators and/or computer algebra systems to solve first order differential equations with Euler's method is assessed through several assigned problems.
Facilitating the building of student conceptual and procedural understanding.	Test questions measure students' understanding of the theories of ordinary differential equations and ability to follow procedures to solve differential equations.
Helping all students build understanding of the discipline including: • Confidence in their abilities to utilize mathematical knowledge. • Awareness of the usefulness of mathematics. • The economic implications of fine mathematical preparation.	How well students can integrate their knowledge from previous courses in calculus and linear algebra with the differential equations material is ascertained through homework and exam questions. Students' understanding of the applications of differential equations to the sciences is measured through assigned problems.
Exploring, conjecturing, examining and testing all aspects of problem solving.	Students skill in exploring and making conjectures when determining which technique might work to solve an ordinary differential equation is assessed through selected homework problems. Their skill in exploring solutions of first order differential equations graphically, through the direction field and making conjectures about the behavior of solutions from the direction field is measured by additional homework problems
Formulating and posing worthwhile mathematical tasks, solving problems using several strategies, evaluating results, generalizing solutions, using problem solving approaches effectively, and applying mathematical modeling to real-world situations.	Homework is used to measure how well students are at comparing solutions to differential equations found using different techniques. For example, students are asked to solve second order linear differential equations using linear independent solutions, using series solutions and using Laplace transforms. Homework is also used to appraise students'ability to use problem solving approaches effectively and to model some real-world applications including growth and decay phenomena, mixing phenomena, and mechanical systems.

Making convincing mathematical arguments, framing mathematical questions and conjectures, formulating counter-examples, constructing and evaluating arguments, and using intuitive, informal exploration and formal proof.	Students' ability to use intuitive, informal techniques to explore the solutions of first order differential equations using direction fields is assessed by assigned problems. Students' skill in making convincing mathematical arguments about the behavior of solutions of differential equations is measured by assignments and exam questions. Students talent at creating proofs relating to the superposition principle and linear independence of solutions is appraised by assigned problems.
Expressing ideas orally, in writing, and visually-, using mathematical language, notation, and symbolism; translating mathematical ideas between and among contexts.	Students' skill in expressing their understanding of the theory and solutions of ordinary differential equations using mathematical language, notation and symbolism is ascertained on written tests and assignments.
Connecting the concepts and procedures of mathematics, drawing connections between mathematical strands, between mathematics and other disciplines, and with daily life.	Sets of homework problems are used to assess each student's ability to make connections between the theory of ordinary differential equations and the solutions to ordinary differential equations, the connections between series solutions and algebraic solutions, and the connections between differential equations and the sciences.
Selecting appropriate representations to facilitate mathematical problem solving and translating between and among representations to explicate problem-solving situations.	Through homework problems and exam questions a student's skill at using the following representations and translations are assessed. 1. Via the Laplace transform, representing a *differential* equation as an *algebraic* equation, lowering the level of difficulty of the problem. 2. Transforming from the *differential* equation representation to the *algebraic* equation representation, solving the *algebraic* equation on the Laplace transform side, and then transforming the solution back to the *differential* equation side.
Mathematical processes including: • Problem solving. • Communication. • Reasoning and formal and informal argument. • Mathematical connections. • Representations. • Technology.	On tests and assignments, students are assessed on their ability to solve problems; communicate their results mathematically; give formal justifications of their reasoning; make connections between theory and example; make connections between solutions found using theory of linear equations, Taylor's series and Laplace transforms; represent solutions to differential equations algebraically, numerically, graphically and with Taylor's series; and use graphing calculators or computer algebra systems to study the behavior of solutions.

Number operations and relationships from both abstract and concrete perspectives identifying real world applications, and representing and connecting mathematical concepts and procedures including: • Number sense. • Set theory. • Number and operation. • Composition and decomposition of numbers, including place value, primes, factors, multiples, inverses, and the extension of these concepts throughout mathematics. • Number systems through the real numbers, their properties and relations. • Computational procedures. • Proportional reasoning. • Number theory.	Students ability to use their number sense, and understand the real number system is measured through their homework assignments and tests.
Mathematical concepts and procedures, and the connections among them for teaching upper level number operations and relationships including: • Advanced counting procedures, including union and intersection of sets, and parenthetical operations. • Algebraic and transcendental numbers. • The complex number system, including polar coordinates. • Approximation techniques as a basis for numerical integration, fractals, and numerical-based proofs. • Situations in which numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social) can be created and critically evaluated. • Opportunities in which acceptable limits of error can be assessed (e.g., evaluating strategies, testing the reasonableness of results, and using technology to carry out computations).	Assessment of a student's ability to solve first order ordinary differential equations numerically, using Euler's method is done through assigned problems and exam questions. Assignments and exam questions are also used to measure a student's skill in using complex numbers in the solutions of linear second order differential equations, with Euler's formula to translate complex solutions into real solutions.

Geometry and measurement from both abstract and concrete perspectives and to identify real world applications, and mathematical concepts, procedures and connections among them including: • Formal and informal argument. • Names, properties, and relationships of two- and three-dimensional shapes. • Spatial sense. • Spatial reasoning and the use of geometric models to represent, visualize, and solve problems. • Transformations and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships. • Coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system. • Concepts of measurement, including measurable attributes, standard and non-standard units, precision and accuracy, and use of appropriate tools. • The structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems. Measurement including length, area, volume, size of angles, weight and mass, time, temperature, and money. • Measuring, estimating, and using measurement to describe and compare geometric phenomena. • Indirect measurement and its uses, including developing formulas and procedures for determining measure to solve problems.	Students' talent in using spatial sense to interpret the behavior of solutions to differential equations and in using area, arc length and volume formulas from calculus is ascertained in assigned homework.

Mathematical concepts, procedures, and the connections among them for teaching upper level geometry and measurement including: • Systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry. • Transformations, coordinates, and vectors and their use in problem solving. Three-dimensional geometry and its generalization to other dimensions. Topology, including topological properties and transformations. • Opportunities to present convincing arguments by means of demonstration, informal proof, counter-examples, or other logical means to show the truth of statements and/or generalizations.	Students' understanding of how to use vectors and matrices in the theory of second order linear equations is appraised via assigned problems and exam questions.
Statistics and probability from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections between them including: • Use of data to explore real-world issues. • The process of investigation including formulation of a problem, designing a data collection plan, and collecting, recording, and organizing data. • Data representation through graphs, tables, and summary statistics to describe data distributions, central tendency, and variance. • Analysis and interpretation of data. • Randomness, sampling, and inference. • Probability as a way to describe chances or risk in simple and compound events. • Outcome prediction based on experimentation or theoretical probabilities.	Not assessed in this course.

Mathematical concepts, procedures, and the connections among them for teaching upper level statistics and probability including: • Use of the random variable in the generation and interpretation of probability distributions. • Descriptive and inferential statistics, measures of disbursement, including validity and reliability, and correlation. • Probability theory and its link to inferential statistics. • Discrete and continuous probability distributions as bases for inference. • Situations in which students can analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.	Not assessed in this course.
Functions, algebra, and basic concepts underlying calculus from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Patterns. • Functions as used to describe relations and to model real world situations. • Representations of situations that involve variable quantities with expressions, equations and inequalities and that include algebraic and geometric relationships. • Multiple representations of relations, the strengths and limitations of each representation, and conversion from one representation to another. • Attributes of polynomial, rational, trigonometric, algebraic, and exponential functions. • Operations on expressions and solution of equations, systems of equations and inequalities using concrete, informal, and formal methods. • Underlying concepts of calculus, including rate of change, limits, and approximations for irregular areas.	A combination of homework problems and exam questions are used to measure students' grasp of the following topics. 1. The basic concepts of calculus. 2. Patterns in families of ordinary differential equations and their solutions. 3. Models of real world situations with differential equations and descriptions of their solutions with functions. 4. Representation of solutions to differential equations algebraically, graphically, and as Taylor's series. How to compare and contrast these representations. 5. Use of the properties of polynomial, rational, trigonometric, algebraic and exponential functions in the solutions to differential equations. 6. Solution of ordinary differential equations using formal algebraic methods, numerically with Euler's method, and graphically with the direction field.

Mathematical concepts, procedures, and the connections among them for teaching upper level functions, algebra, and concepts of calculus including: • Concepts of calculus, including limits (epsilon-delta) and tangents, derivatives, integrals, and sequences and series. • Modeling to solve problems. • Calculus techniques including finding limits, derivatives, integrals, and using special rules. • Calculus applications including modeling, optimization, velocity and acceleration, area, volume, and center of mass. • Numerical and approximation techniques including Simpson's rule, trapezoidal rule, Newton's Approximation, and linearization. • Multivariate calculus. • Differential equations.	A mix of homework problems and exam items are used to assess students' achievement in the following areas. 1. Knowledge of the concepts of calculus, including limits (from an intuitive viewpoint), tangents, derivatives and series. 2. Use of modeling to solve real-world problems such as growth and decay phenomena, mixing phenomena, and mechanical systems. 3. Use of calculus techniques to find derivatives and integrals. 4. The solution of first order linear and non-linear ordinary differential equations. 5. The solution of second order linear ordinary differential equations using the techniques involving the theory of linear equations, Taylor's series and the Laplace transform.
Discrete processes from both abstract and concrete perspectives and to identify real world applications, and the mathematical concepts, procedures and the connections among them including: • Counting techniques. • Representation and analysis of discrete mathematics problems using sequences, graph theory, arrays, and networks. • Iteration and recursion.	Not assessed in this course.
Mathematical concepts, procedures, and the connections among them for teaching upper level discrete mathematics including: • Topics, including symbolic logic, induction, linear programming, and finite graphs. • Matrices as a mathematical system, and matrices and matrix operations as tools for recording information and for solving problems. • Developing and analyzing algorithms.	Students' understanding of the theory of linear differential equations by the use of matrices is assessed through assigned problems.